Where $P$ was an index of manufacturing production, $L$ was labor input (index of average number of wage earners in manufacturing), $C$ was the capital input (index of fixed capital in manufacturing) and $b$ and $k$ were constants to be found by the classical least squares regression method.

In this form the function assumes that the sum of the exponents is unity, and hence that there existed constant returns to scale. In later studies this restriction was removed and more refined variations on data used, with the function assuming the form

$$P=bL^kC^j$$

so that $k+j$ could be equal to, less than, or greater than unity. The studies of Professor Douglas and his associates, as well as others, tended to arrive at estimates of $k+j$ close to or equal to unity, though with some tendency toward diminishing returns. (28)

This later form of the Cobb-Douglas function was chosen as a starting point and modified in light of subsequent research results and of the changes in availability of data. Five modifications were made:

(1) Labor input, expressed in man-hours, was split into two components, first, a secular component ($Lp$) based on longrun or secular changes in the rates of participation of the population in the labor force and secular trends in hours of work and second, a cyclical component, the ratio of actual labor input to the potential labor input $\left(\frac{La}{Lp}\right)$.

(2) Capital (measured as a gross stock in constant prices) was used in the form of a ratio of the stock of capital to the potential labor input ($K/Lp$).

(3) The influence of changes in technology on productive efficiency was captured in part by inserting a new variable, the average age of the capital assets surviving in the capital stock in each year ($k$). The lower the average age of the capital stock, the closer the productive process was assumed to approximate the latest, most efficient techniques; the older the average age, the less the stock incorporates the most efficient known techniques. This is a measure of qualitative changes in the capital stock ($K$).

(4) A time trend ($t$) was introduced to measure the influence of all those intangibles which could not be measured directly. This is a proxy variable, accounting in this case for technical and other changes, or in technical terms for any kind of shift in the production function.

(5) An index ($X$) was introduced, so constructed as to measure the influence of change in demand on the total productivity of the inputs. This index ($X$) reflects changes in the composition of demand between products and services requiring varying amounts of labor and capital per unit.

Inserting these changes in the classic Cobb-Douglas production function, a formula was obtained in the following form, where $O_a$ represents output:

$$O_a=A.Lp.\left[\frac{La}{Lp}\right]^a.\left[\frac{K}{Lp}\right]^b.k^c.x.d^t$$

The formula was tested against statistical data by least squares, by procedures outlined along with the results in chapter III. The formula had to be modified further during the fitting process.