and the mix variable (X); after which the optimum fit could be found by least-squares. The mix variable (X) by definition has a coefficient of 1 since it is a measure of changes in productivity of labor and capital combined, assuming all input efficiencies constant but the composition of demand variable.

It was clear at an early stage that a linear form for the cyclical variable $\left[\frac{La}{Lp}\right]$ was not adequate and a nonlinear form had to be adopted. This was parabolic in logs—a result consistent with the economic theory of production with fixed supplies of factors, fixed technology, fixed mix of demand, and variable rates of use of inputs. The final equation therefore used both $\log \left[\frac{La}{Lp}\right]$ and $\left[\log \frac{La}{Lp}\right]^2$ as variables.

The experiments clearly pointed to a nonlinear form also for the age of capital (k) but with a negative curvature in the relevant range. This agreed with expectations. An increase in age of the capital stock (k) implies that, other things being equal, the technological efficiency of the capital stock is reduced below what it would be if capital, on the average, were younger and approximated more nearly the best production techniques known at the time. The curvature also was reasonable since the older the capital stock becomes, on the average, the smaller would be the proportionate effect on efficiency of a further aging of the capital stock. Consequently, the age of capital entered the final equation as both $\log k$ and $[\log k]^2$.

The final equation

With these points arrived at, the final fit was arrived at on the seventh run.

The final equation chosen on the basis of the many tests was:

$$\begin{aligned}\log O_m = & -5.43104 + \log L_p + .9104 \log (L_a/L_p) \\ & -3.39[\log (L_a/L_p)]^2 + .35 \log (K/L_p) - 5.6411 \log k \\ & +10.356(\log k)^2 + X + .00884t\end{aligned}$$

N.B.—O_m is the predicted value of gross national product and is the equation's estimate of O_a. The relation of O_m to O_a is shown in column 3 of table 2.

N.B.B.—The average age of capital k has been measured in decades when fitting the equation because of the scaling problem introduced by a squared term if coded input is used.

The equation provided a high degree of explanation of the variations in output (Oa). The R^2 was .9898 and the closeness of the fit can be seen from the chart V, where the log of the predicted value is plotted vertically against the log of the actual output on the horizontal axis. It will be noted that the dots, representing the various years, are clustered closely around the 45° line which represents a perfect correspondence between predicted and actual output. All of the variables were statistically significant as measured by their standard errors (37).

How sensitive are predicted values to variations in the equation? A test was made by making predictions of potential output for *1959* and *1975* using eight different versions of the equation and similar values for the inputs. The results showed a maximum deviation from the output computed by the final equation equal to about 12 percent, with six of the eight equations staying within 2 percent. Seven of the sixteen predictions fell within 1½ percent of the final equations results