

Exact quantification of relationship, as well as assessment of its statistical significance requires the full regression analysis presented in Appendix E. From this analysis we have drawn the regression line fitted to the points. The equation is:

$$(I_B) \text{ Industry Return} = 8.6 + 0.038 (\text{Industry Risk})$$

The b_2 ($=0.038$) coefficient is highly significant ($F=50$) at 57 degrees of freedom. The correlation coefficient (R) is .68 while R^2 equals .46.

Figure 2 portrays the same relationship, with Return and Risk measured at market value.

A glance at the graph or the equation (I_M) reveals much the same story:

$$(I_M) \text{ Industry Return} = 14.4 + .007 (\text{Industry Risk})$$

In this equation b_2 is again highly significant having an F test value of 82 with 57 degrees of freedom. The correlation coefficient equals .77 (out of a possible 1.00) while R^2 reached 0.59—most impressive for a basically cross-sectional analysis.

Now that the hypothesized relationship between risk and return has been statistically validated, we may turn to questions concerning individual industries and their relation to the normal risk/return pattern. Given that an industry has a higher than average rate of return, it is both meaningful and possible to ask whether its risk is proportionately higher than average risk. For example, in Figure 1 the pharmaceutical industry has one of the highest book rates of return as well as one of the highest risks. Using the regression statistics we may test whether its particular risk/return point could reasonably be generated by the economic mechanism described by the regression line. Using a two standard error test—i.e., greater than 95% confidence limits—we can conclude that the point does belong to the normal pattern.¹⁰

In an attempt to examine other dimensions of risk we expanded our original model to include temporal variances as well as the spacial variance first used to measure risk. The yearly variance of the industry's rate of return about its temporal average was added first:

$$(I') \text{ Industry Return} = a + b_2 (\text{Industry Risk}) + b_3 (\text{Industry Temporal Variance})$$

In both equations I'_B and I'_M there is some improvement in R^2 from 0.46 to 0.51 and from .59 to .61, respectively. This improvement was at the cost of a reduction in degrees of freedom as well as the value of the F statistic. Including an average of the individual companies' temporal variance added little or no independent or partial explanatory power.

Although we are primarily concerned with explaining industry rates of return, some time was devoted to analyses of individual company returns. The basic model used was:

$$(II) \text{ Company Return} = a + b_1 (\text{Company Risk}) + b_2 (\text{Industry Risk})$$

Company risk is the standard temporal variance used by most researchers. While it yields the expected results (b_2 positive and significant) for market rates of return, Model II yields a significantly negative value of b_2 for book rate of return. We attribute this negative coefficient to the high degree of auto-correlation found book value statistics. (In Section III we rejected the temporal variance as a measure of risk on theoretical grounds related to auto-correlation.)

In summary, we have seen that our hypothesized measure of industry risk has been statistically validated for both book and market measures of industry

¹⁰ Mathematically this finding is derived from the limits of prediction formula (cf. Miller & Freunds, *Probability and Statistics for Engineers*, p. 235):

$$(a + b_2 x_0) \pm t_{\alpha/2} S_e \left[1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{S_{xx}} \right]^{1/2}$$

where $t_{\alpha/2} = 2$. Setting x_0 to 74 we get a y range of 19.4 to 10.2. The observed y value of 17.6 falls well within this range.