TABLE 4
SIMPLE-DISCOUNT METHOD: BREAKDOWN OF PRESENT VALUE OF FUTURE INSTALLMENTS AND AMOUNT OF DISCOUNT, BY MONTHS, BASED ON EXAMPLE[1]

Months before installment is due	*Present value of future installments*	*Discount or finance charge*[2]	*Amount of installment payable at due date*
Number	*Dollars*	*Dollars*	*Dollars*
1	19.72	0.28	20
2	19.44	.56	20
3	19.17	.83	20
4	18.89	1.11	20
5	18.61	1.39	20
6	18.33	1.67	20
7	18.06	1.94	20
8	17.78	2.22	20
Total	150.00	10.00	160

[1] Discount rate 16⅔ per cent a year; installment term, 8 months; periodic payment, $20.
[2] Collected as installments are paid. Figures in this column are the same as for the direct-ratio method except that they are in reverse order.

carrying charge I by $1/m$ times the sum of the assumed beginning-of-the-month balances, starting with pn for the first month. This sum also includes a balance of $pn-p$ for the second month, $pn-2p$ for the third month, and $pn-p(n-1)$ for the last month. The sum of these balances is $pn(n+1)/2$. Multiplied by $1/m$, the result is $pn(n+1)/2m$, and

$$I \div \frac{pn(n+1)}{2m} = \frac{2mI}{pn(n+1)}.$$

A discount rate equivalent based on assumed beginning-of-the-month balances must include the finance charge in the first month's balance, that is, the beginning month's balance must include or be the sum of all future payments, because discount is figured on maturity values rather than present values.

SIMPLE INTEREST

The viewpoint might be taken that the seller has "loaned" the buyer the difference between the cash price and the down payment. This amount $(C-D)$ might be considered as the sum of the present values, at simple interest, of the respective future payments. The question, then, is . . . At what simple-interest rate would these amounts "grow" to the amount of each monthly payment by the time it becomes due? In the example

$$\$150 = 20\left[\frac{1}{1+i/12}\right] + 20\left[\frac{1}{1+2i/12}\right] + \cdots 20\left[\frac{1}{1+8i/12}\right].$$

By substituting successive values of i in the simplified equation, the summation of the reciprocal series is found to be 0.6251607 when $i=0.180$. For $i=0.181$, it is found to be 0.6249478. So, by interpolation, the required value of i is 0.180756, or 18.0756 per cent.

In table 5, the $150 "borrowed" is made up of the amounts shown in column 3. The $19.70 for the first month "grows" to $20 at the end of the first month at an annual rate of 0.180756, or a monthly rate of 0.015063. At that time, it is paid to the "lender." The $19.43 "grows" to the $20 that is paid at the end of two months, and so on.[8] The distribution of the finance charge, by months, is shown in column 4.

The annual rate at simple interest also may be verified by dividing the finance charge ($10) by one-twelfth of the sum of the beginning-of-the-month balances ($663.87).

From the preceding discussion, it can be seen that the computation of an equivalent annual rate at simple interest is very complex. A rate so computed is comparable to one computed on money borrowed from a lender who charges interest on the

[8] If interest at 0.015063 per month were collected on the unpaid balance, compound interest would be charged and the last payment of $20 would lack 36 cents of canceling the debt. It takes a monthly rate of only 0.01456 to exactly amortize the $150 in 8 months at $20 a month. (See small-loan and present-value methods.)