

discounted value of his pensions at the time of retirement is area P less area D . With interest available, he has to save, or tax himself, at a lower rate than with no interest because of the accumulation of interest on his savings and the discounting of the value of his pension.

Inspection of the charts suggests the break-even point. The advantage of the collective pay-as-you-go rate is just offset when the interest rate is equal to the rate of growth of population.⁵

If the interest rate exceeded the rate of population growth, the individual would fare better by doing his own saving than he would under a collective pay-as-you-go insurance system. The question of whether there would be a positive interest rate in an economy in which wages remained constant is a complex one. In a more elaborate model, Professor Samuelson has shown that under conditions similar to those assumed above, the interest rate will indeed be *determined* by the rate of population growth.⁶

THE INSURANCE TAX COMPARISON IN A PROGRESSING ECONOMY

We have shown that the collective pay-as-you-go tax rate is independent of the rate of growth of wages. However, the rate of growth of wages (or "productivity") affects the individual's calculation of his required rate of saving. With a growing wage rate, he will have to tax himself more in every year before retirement in order to provide a pension that grows with the level of wages from his year of retirement. If he is to keep up with the Jones' after his retirement, he will have to tax himself at a higher rate over his working life.⁷

Under an increasing-wage assumption, the additional saving required will more or less offset the additional value obtained from interest—depending on the extent to which the rate of interest exceeds to the rate of growth of wages.

As Henry Aaron has shown in a slightly different formulation,⁸ the collective pay-as-you-go rate will be equal to the individual's "actuarial" rate, where the interest rate is approximately equal to the sum of the rate of growth of population and the rate of growth of wages. This relationship is shown by a comparison of charts A-1 and A-3. In chart A-3, an increasing wage assumption is illustrated by the rising wage and tax curve. For the sake of direct comparison with chart A-1, interest is shown on chart A-3 accumulated graphically from year a_2 to year a_1 . The interest shown in area I_1 is the amount that would be accumulated if the interest rate were just equal to the rate of growth of wages. The interest shown in area I_2 plus area I_1 is the amount that would be accumulated if the interest rate were just equal to the sum of the rate of growth of wages and the rate of growth

⁵ Let T_m = the tax paid in the last year of working life, T' = the total taxes paid over the individual's working life, and $r = 1 +$ the rate of interest. Then: $T' = T_m(1 + r + r^2 + \dots + r^{m-1})$. Similarly, let w_m = the wage paid in the last year of working life, and P' = the discounted value of the individual's pension payments at the time of his retirement. Then: $P' = w_m(r^{-1} + r^{-2} + \dots + r^{-m})$. From these equations and those in footnote 3, it follows that $T' = P'$ when the interest rate is equal to the rate of population growth.

⁶ Paul A. Samuelson, "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, vol. 46, December 1958, pp. 467-482.

⁷ Even if he chose only to provide himself a pension equal to the average wage when he retired, he would also have to save more in his working years when his wage averaged less than the wage in his last working year.

⁸ *Op. cit.*, pp. 373, 374.