

type and is concerned only with per capita taxes and per capita benefits. Let—

$\bar{E}_t$ = average (mean) real earnings in year t

$b_t = k\bar{E}_t$ = average real benefit in year t (where k is fixed ratio of current $\bar{E}_t$ to $\bar{E}_0$)

W_t = total earning (taxpaying) population

R_t = total old-age beneficiary (largely retired) population

T_t = average real tax in year t (including both employer and employee contributions)

Assuming pay-as-you-go financing and an exponential growth rate r and $\bar{E}_t$,

$$\bar{T}_t = \frac{k\bar{E}_t R_t}{W_t} = \frac{k\bar{E}_0 (1+r)^t R_t}{W_t} \quad (1)$$

Given k and projections of $\bar{E}$, R , and W , the projected average tax can be obtained and accumulated along with imputed interest.

The present value of the average benefit stream beginning at age 65²² may then be compared to the accumulated tax. Let—

b_n = average benefit during the year at age n

B = present value of the average benefit stream at age 65

P_n = probability of reaching age n , given that one has reached age 65

L = length of working career

The discounted present value of the still-growing benefit stream is given by—

$$B = \sum_{n=65}^{\infty} \frac{b_{65}(1+r)^{n-65}}{(1+i)^{n-65}} \cdot P_n \quad (2)$$

However, the first annual benefit at age 65 is—

$$b_{65} = k\bar{E}_0(1+r)^L$$

Therefore, equation (2) may be rewritten as—

$$B = k\bar{E}_0 \sum_{n=65}^{\infty} \frac{(1+r)^{L+n-65}}{(1+i)^{n-65}} \cdot P_n \quad (3)$$

It is apparent from equation (1) that the accumulated tax must be directly proportional to (a multiple of) $k\bar{E}_0$ which is the average benefit in the initial year. Since equation (3) shows this to be true also of the present value of benefits at retirement the initial benefit factor cancels out in computation of the tax-benefit ratio; since the latter is therefore independent of the initial benefit, the accumulated tax can be stated generally as a multiple of the starting benefit level, and no estimate is needed of the latter. Under the assumptions of the model the ratio of the accumulated average tax to the present value of average benefits at age 65 depends only on the assumed values of the growth

²² Estimates will be given for this typical retirement age only, although the Social Security Administration puts the average retirement age at 67. This avoids the complexities of the "earnings test" but qualifies the results since later retirement leads to more taxes and less benefits. On the other hand it seems likely that the standard retirement age will eventually be lowered, so that age 65 may be a plausible compromise.