

criterion is susceptible to various interpretations. As indicated earlier, we use it to mean benefits of each cohort are equal to its contributions plus interest.

C. *Solvency*.—The system has to generate sufficient contributions to pay all benefits and administrative costs when these are due. It does not accumulate reserves or incur debts in the long run.

D. *Administrative simplicity*.—Contributions and benefits should be easy to understand, compute, and predict.

E. *Stability*.—The contribution rates should be sustainable in the long run, which means that the system should not require or imply a tax rate that increases beyond reasonable limits at any future time.

Certain other assumptions and characteristics of our models may be stated at this point. We take the macroeconomic variables, such as the growth of income and of population as given, and we do not take into account any impact of social security on these variables. Our models are deterministic in the sense that all relevant aspects of the future are assumed to be known except benefits and contributions.

III. A MATHEMATICAL MODEL OF A PERMANENTLY SELF-SUSTAINING SYSTEM

In this section we develop a continuous model of a social security system. Although this model has serious limitations, it leads to useful insights and hypotheses on the nature of the set of solutions to our problem of deriving a social security system that satisfies the constraints A through E.

We assume the following data are given :

- $g \equiv$ The working lifetime.
- $w(t, n) \equiv$ The wage rate at time t for workers that have been in the work force for n time units. This function is defined and continuous for all $t \geq 0$ and all n in the interval $0 \leq n \leq g$.
- $D(t) \equiv$ The rate of entry into the work force at time t . This function is defined and continuous for all $t \geq 0$.
- $\alpha \equiv$ An interest rate used to relate social security contributions to benefits.

The requirements B, C, and D are implemented as follows :

B. *Equity*.—We set the lump-sum payment to the cohort that retires at time t equal to the total contribution made by this cohort during its working lifetime plus interest continuously compounded at the rate α .

C. *Solvency*.—We require that the total (tax) revenue of the social security system at time t be equal to the lump-sum benefit paid to the cohort that retires at time t .

D. *Administrative simplicity*.—We require that the social security tax rate at time t , which we define as $r(t)$, be charged against all income from work at time t .

These assumptions, together with specified initial conditions are enough to determine the social security tax rate $r(t)$ for all $t \geq g$. We note the specific absence of the social adequacy condition among those above that are sufficient to determine the tax rate schedule $r(t)$. We note also that the tax rate schedule $r(t)$ is sufficient, together with the equity condition, to determine the schedule of lump-sum benefit payments.