We assume that the rate of entry into the work force, $D(t)$ satisfies the following relation:

$$\frac{dD(t)}{dt}=\pi D(t). \tag{3.4}$$

This implies

$$D(t)=D_0e^{\pi t}, \tag{3.5}$$

where D_0 is a given constant. The parameter π is also the percentage growth rate of the labor force.[1]

We also assume that wages are governed by a constant percentage growth rate; i.e.,

$$\frac{\partial w(t,x)}{\partial x}=\gamma w(t,x),$$

which implies

$$w(t,x)=e^{\gamma t}.w(x) \tag{3.6}$$

where $w(x)$ is a given positive and continuous function[2] of x for $0\leq x\leq g$.

Now, after substituting into (3.3) on the basis of (3.5) and (3.6) we get

$$r(t)=\frac{D_0e^{\pi(t-g)}\int_0^g w(x)e^{\gamma(t-g+x)}r(t-g+x)e^{\alpha(g-x)}ds}{\int_0^g w(x)e^{\gamma t}D_0e^{\pi(t-x)}dx}. \tag{3.7}$$

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[1] Define:

$P(t)$ = Total work force at t.
$B(t)$ = Cumulative entries in the work force at t.
$E(t)$ = Cumulative departures from the work force at time t.

Then

$$P(t)=B(t)-E(t),$$

and since

$$E(t)=B(t-g),\ P(t)=B(t)-B(t-g).$$

We assume

$$\frac{dB(t)}{dt}=\pi B(t).$$

This implies: (i)

$$B(t)=B_0e^{\pi t},$$

and

$$\frac{dB(t)}{dt}=\pi(B_0e^{\pi t})=(\pi B_0)e^{\pi t};$$

since

$$D(t)=\frac{dB(t)}{dt}$$

we have

$$D(t)=D_0e^{\pi t}$$

where

$$D_0=\pi B_0;$$

and (ii)

$$\begin{aligned}\frac{dP(t)}{dt}&=\frac{dB(t)}{dt}-\frac{dB(t-g)}{dt}\\&=\pi B(t)-\pi B(t-g)\\&=\pi(B(t)-B(t-g))\\&=\pi P(t).\end{aligned}$$

[2] $w(x)$ is a function that gives the wage rate for each cohort, x, where $0\leq x\leq g$. This is a profile of wages versus time in the work force. The entire profile inflates at the percentage rate γ. One of the results of this model is that the stability of the social security system does not depend on the shape of the profile $w(x)$.