forming a lump-sum benefit at the instant of retirement into a time path of benefits for each member of the cohort over his retirement years. Then we assume it is possible to determine whether the time path of benefits is, for each member, socially adequate. If so, the lump-sum benefit is socially adequate; otherwise it is not. We are not persuaded that the minimum socially adequate lump-sum benefit should increase (with time) at the percentage rate $\gamma+\pi$. Indeed, very casual consideration of such dynamic factors as changing life expectancy suggests that the time path of the minimum benefit would not have a natural harmonious relation to the growth of total wage income. Accordingly we would expect shortrun situations in which the benefits generated by our model are not socially adequate.[1] As a means to dealing with this, and other shortrun problems, we propose a linear programing model in the next section.

APPENDIX TO SECTION III

In this appendix we develop a simple relation between a set of initial conditions and a resulting equilibrium constant tax rate.

Consider the problem posed by the following three requirements:

(i) During a period $0\leqq t<g$ we require that each cohort retiring at time t be awarded a benefit $V(t)$ equal to the interest-weighted sum of the cohort's contributions from 0 to t plus an amount $f(t)$. The function $f(t)$ is positive and continuous for $0\leqq t<g$.

(ii) We permit the social security system to borrow and lend at the rate ρ during the period $0\leqq t<g$ and only require that no debt or loan should be outstanding at $t=g$.

(iii) We seek a constant tax rate r to hold for this period (i.e., $r=r(t)$, $0\leqq t<g$) that is consistent with (and, in fact, determined by) requirements (i) and (ii).

This problem is easily solved. We have, from (i)

$$V(t)=f(t)+rD(t-g)\int_0^t w(t-g+x,x)e^{\alpha(g-x)}dx \qquad (3A.1)$$

or defining

$$B(t)=D(t-g)\int_0^t w(t-g+x,x)e^{\alpha(g-x)}dx,$$

we have

$$V(t)=f(t)+rB(t).$$

Let revenue at t be denoted by $K(t)$. Then

$$K(t)=r\int_0^g w(t,x)D(t-x)dx$$

and defining

$$A(t)=\int_0^g w(t,x)D(t-x)dx,$$

we have

$$K(t)=rA(t).$$

[1] We recognize that it is possible to generate stable systems with tax rates and benefits sufficiently high to insure minimum socially adequate benefits at all points in time. The problem of solving for a tax rate that is optimal—taking into account social adequacy and the various disutilities induced by higher tax rates—is beyond the scope of this paper. (See above, p. 2.)