From (ii) we have

$$\int_0^g [K(t)-V(t)]e^{\rho(g-t)}dt=0.$$

Hence

$$\int_0^g [rA(t)-f(t)-rB(t)]e^{\rho(g-t)}dt=0$$

$$r\int_0^g [A(t)-B(t)]e^{\rho(g-t)}dt=\int_0^g f(t)e^{\rho(g-t)}$$

$$r=\frac{\int_0^g f(t)e^{\rho(g-t)}dt}{\int_0^g [A(t)-B(t)]e^{\rho(g-t)}dt}. \qquad (3A.2)$$

We can draw several conclusions—heuristically at least—from this result:

(*a*) If $w(t,x)$ and $D(t)$ are specified by (3.5) and (3.6) and if $\alpha=\gamma+\pi$ than it is evident from (3.8) that setting $r(t)=r$ as defined by (3A.2) for $0\leq t<g$ will lead to r as a perpetual constant tax rate. Thus by slightly relaxing the solvency requirement we see that the imposition of exogenous requirements on the system (i.e., the function $f(t)$, $0\leq t<g$) can be accommodated without fluctuations in the tax rate. (3A.2) provides a relationship linking the resultant tax rate to the exogenous stimulus $f(t)$.

(*b*) The assumptions of this analysis are not operationally offensive, particularly if $\rho=\alpha=\pi+\gamma$. It would seem reasonable to substitute a moderate program of short-term borrowing and lending, especially at no net cost, for fluctuating tax rates.

(*c*) The requirement imposed in (i) by $f(t)$ is equivalent to the requirement imposed by starting the system at $t=0$ and assuming that the system as a given "debt" or obligation to each working cohort at $t=0$. Let $0(0,x)$ be the obligation of the system at time zero to the cohort in its xth year in the work force. Defining $f(t)\equiv e^{\alpha t}\cdot 0(0,g-t)$ we can use (3A.1) to determine $V(t)$ in accordance with the usual rule that benefits equal payments plus interest at the rate α.

IV. A Linear Programing Model

There are several reasons, in addition to those cited in section III, for using linear programing to solve for a social security system. Linear programing considerably extends our ability to solve problems with a wide variety of constraints, including the requirement for minimum retirement income that was "unrepresented" in the calculus model of section III. We can add other constraints besides those that have occupied our attention thus far. We can express a variety of interpretations of the constraints we have considered; solvency, for example, does not necessarily require a continuous exact balance of receipts and expenditures. This ability to vary the mathematical representations of a given category of constraint provides a method of testing, at worst by computational experiments, to determine whether certain characteristics of the system are caused by essential or incidental properties of a given constraint.