The linear programing model will also accommodate arbitrary patterns of wage rate and labor force growth, in contrast to the assumption of smooth exponential growth paths that was required to extract informative results from the model of section III. Thus linear programing adapts well to the mathematically messy details of shortrun empirical data or predictions. More fundamentally the linear programing model provides a framework in which problems of fact and value can be appropriately segregated for the purposes of making an informed resolution of conflicting interests, and appropriately integrated for the purposes of achieving an optimal, articulated plan of action.

We now discuss the formulation of a social security problem. Our goal here is to develop a formulation that is conceptually as close as possible to the model of section III. We shall subsequently discuss some of the many possible elaborations of this elementary model.

In contrast to the calculus model of section III, the linear programing model will deal with a fixed span of time from year 1 through year T. This immediately raises a seemingly new problem: How do we deal with obligations existing at $t=1$ to people in the work force at that time? And how do we account for the unredeemed obligation to people in the work force at the terminal planning date T? Explicit means of dealing with these problems will be discussed presently. In general terms these problems had their counterparts in the calculus model; the first problem is analogous to specifying $r(k)$ for $0 \leqq k < g$ and the second is analogous to the stability criterion.

Formally a linear programing problem may be characterized as follows: Minimize

$$Z-\sum_{J}^{n} c_j x_j \tag{4.1}$$

subject to

$$x_j \geq 0 (j=1, \ldots, n) \tag{4.2}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i (i=1, \ldots, m) \tag{4.3}$$

The inequality in (4.3) is only illustrative and could be oppositely oriented or could be an equality. The x's are the variables and the a's, the b's, and the c's are the data or the parameters of the problem. Thus the social security tax rates $r(t)$, $t=1, 2, \ldots, T$, and the benefits $V(t)$, $T=1, 2, \ldots, T$, will play the role of the x's. And the requirements of the system (A) through (E) on pages 139 and 140 will be expressed as linear restrictions such as (4.3) on the variables.

It is not obvious that the minimization required in (4.1) is pertinent, since there was no minimization involved in the calculus model of section III.

A minimization problem arises, in fact, out of the already cited deficiency of the calculus model: it may not satisfy all the requirements of a social security system. Since the calculus model does satisfy some requirements of the system and these requirements are sufficient to determine a solution, it would not appear generally possible (once the initial conditions are determined) to simultaneously satisfy all requirements. Hence it is appropriate to attempt minimization of a weighted sum of the (inevitable) violations of the various constraints.