Linear programing provides a completely natural and effective means of implementing such a minimization goal. Let a typical restriction be represented by $\sum_j a_j x_j \geq b$. This constraint is routinely converted into an equivalent equation by adjoining a slack variable s: $\sum_j a_j x_j - s = b$. s is required to be nonnegative and can be interpreted as a measure of the overfulfillment of the constraint. It is also possible to adjoin another variable ω to the constraint: $\sum_j a_j x_j - s + \omega = b$. ω is also constrained to be nonnegative and is a measure of the amount by which the solution fails to satisfy the constraint. Variables such as ω are called artificial variables. An artificial variable may be adjoined to each constraint in the model. To minimize the appropriate sum of constraint violations one specifies the constants c_j in (3.1) as follows: Only the artificial variables have nonzero c_j coefficients; and the c_j associated with an artificial variable is a measure of the relative cost (subjective or objective) of a unit violation of the constraint associated with the artificial variable.[1]

To formulate and solve a linear programing version of the social security problem we assume that the following array of known constants is available as basic data:

$w(t,x) \equiv$ Annual wage in year t for the cohort in its xth year in the work force ($t=1, 2, \ldots, T$: $x=1, 2, \ldots, g$).

$D(t) \equiv$ Number of workers in the cohort that enters the work force in year t ($t=1, 2, \ldots, T$).

$y(t,x) \equiv w(t,x)\, D(t-x+1) =$ total wage income in year t for the cohort in its xth year in the work force ($t=1, 2, \ldots, T$: $x=1, 2, \ldots, g$).

The variables for the problem are:

$r(t) \equiv$ The social security tax rate charged in year t against wage income ($t=1, 2, \ldots, T$).

$V(t) \equiv$ The lump-sum benefit payment made to the cohort that retires at the end of year t ($t=1, 2, \ldots, T$).

The model includes the following types of constraints: equity constraints, solvency constraints, social adequacy constraints, and a constraint governing terminal conditions. In the definition of the variables we have implicitly included another constraint reflecting a desire for a simple tax rate structure, by defining tax rate variables as we have, we rule out the possibility of charging different tax rates against different cohorts. We discuss this option later.

We assume that the planning period, $T > g$, the working lifetime.

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[1] The final specification of the weights $c_1, c_2 \ldots c_m$ is psychologically easier and procedurally much more complex than our discussion here suggests. Contrary to appearance, the values of $c_1, c_2 \ldots c_m$ can be specified without forcing the decisionmaker into an a priori specification of his utility function over the variables $\omega_1, \omega_2, \ldots, \omega_m$. And the linear form of Z is less restrictive than it appears. Specifying the parameters c_i that define Z amounts to provisionally specifying a tangent hyperplane to an indifference surface rather than the surface itself. A sequence of such provisional specifications leads to final specification of the tangent hyperplane at a utility maximizing point. For details see Bertil Näslund, "Decisions Under Risk (Economic Applications of Chance-Constrained Programing)," doctoral dissertation, Carnegie Institute of Technology, November 1964.