Defining

$$0(T,\ x)=\sum_{k=0}^{x-1}y(T-k,\ x-k)\alpha^k r(T-k),\ (x=1,\ 2,\ .\ .\ .\ ,\ g-1) \quad (4.8)$$

as the obligation of the system to the cohort that is in its xth year in the work force at time T, the total unredeemed obligation at time T is given by

$$0(T)=\sum_{x=0}^{g-1}O(T,\ x). \quad (4.9)$$

Our constraint is simply

$$0(T)\leq H. \quad (4.10)$$

This raises two questions:

(i) How is H determined?

(ii) How does this constraint balance the interests of those that retire before and after year T?

Before answering these questions we shall note that without any constraint such as (4.10) the likely result would be a solution implying an unjustifiably large $0(T)$, since the benefits derived from a large $0(T)$ would be enjoyed before T, within the scope of the problem, while the costs of redeeming the debt $0(T)$ would be borne after T, beyond the horizon of the problem. Thus we have bounded $0(T)$ from above.

Analysis of the calculus model indicates that $0(T)$ grows at the percentage rate α. This is consistent with longrun stability of the social security tax rate, if and only if α is equal to the percentage growth rate of total wage income. Thus we would recommend setting

$$H=K\alpha^T \quad (4.11)$$

where K is the obligation of the system at $t=1$; i.e.,

$$K=\sum_{x=1}^{g}0(1,x)=0(1). \quad (4.12)$$

Without undertaking an analysis that is beyond the scope of this paper, we cannot make definitive claims for (4.11) and (4.12) as a means of preserving intergenerational equity. The idea behind (4.10), (4.11), and (4.12) is that a unique constant tax rate r_1 is consistent with the obligtion $0(1)$ at the beginning of the period; and we assume that r_1 is normative in this sense: an obligation $0(T)$ at T, that is consistent with r_1 represents an optimal balancing of the interests of those that retire before and after T. We emphasize again that determination of an optimal longrun average rate r_1 or an optimal time path $r(t)$ is a problem we do not attempt to solve here. And determination of appropriate terminal conditions must follow from the explicit or implicit solution of such a problem. Hence our terminal constraint is only conditionally normative.[1]

[1] The analysis of the appendix to sec. III is relevant here in two respects. First, the general nature of the connection between the obligation at t, $0(t)$, and a tax rate r for succeeding periods is explored there. Second, the results of that section suggest that (exponential) weights might be appropriately included on the right side of (4.9).