

APPENDIX III

PAY-AS-YOU-GO FINANCING VERSUS ADVANCE FUNDING

A. ALGEBRAIC ANALYSIS

An employer who contributes \$1 now to his pension fund will thereby be able to discharge a benefit payment of $\$(1+j)$ 1 year later, or more generally $\$(1+j)^n$ n years later, where j is the annual net yield of the pension fund. A taxpaying employer also gets a \$1 deduction now for income tax purposes.

On the other hand, if the employer does not fund in advance, he keeps his \$1, less taxes at rate t (assuming the employer would contribute from current earnings), and can invest it in his business or elsewhere at an annual rate of return of i before taxes, or $i(1-t)$ after taxes. When, n years later, he makes the benefit payment of $\$(1+j)^n$, the resulting deduction reduces his taxes by $t(1+j)^n$.

The employer's choice of financing method has two important consequences:

(1) *For the employer.*—Making the comparison at time n , pay-as-you-go financing is better for the employer if the amount gained by not funding in advance; i.e., $\$(1-t)[1+i(1-t)]^n$ is more than the net cost of the benefit; i.e., $\$(1-t)(1+j)^n$. Advance funding is preferable for the employer if and only if $j > i(1-t)$.

(2) *For the Federal Government.*—Suppose money is worth k percent per year to the Federal Government. Then, making the comparison at time n , pay-as-you-go financing is better for the Federal Government if the value of the taxes on the original \$1 and on its subsequent earnings; i.e.,

$$\begin{aligned} & \$t(1+k)^n + \$ti(1-t) \sum_{r=0}^{n-1} [1+i(1-t)]^r (1+k)^{n-r-1} \\ & = \$t(1+k)^n + \$ti(1-t) \frac{[1+i(1-t)]^n - 1 + k}{[1+i(1-t)] - 1 + k} \end{aligned}$$

is more than the tax loss at time n , which is $\$t(1+j)^n$.

Advance funding is preferable for the Federal Government if and only if

$$t(1+j)^n > t(1+k)^n + it(1-t) \frac{[1+i(1-t)]^n - (1+k)^n}{[1+i(1-t)] - (1+k)}.$$

(NOTE.—Formulas developed by Harrison Givens, Jr., associate actuary of the Equitable Life Assurance Society.)