

present value of his lifetime income, earnings plus pension, will be higher if no reserves are accumulated than if they are. This fact holds for every person. Consequently, *for the Nation as a whole, the present expected value of the sum of real net lifetime receipts is greater when reserves are not accumulated than when they are, although the national income at each point in time is unaffected.* Has a fallacy crept in?

From an insurance equity standpoint it has. The fact that the same pension is paid under the pay-as-you-go assessment system and the reserve system, although premiums are higher in the latter case than in the former, means simply that a higher interest rate is implicitly being paid on contributions under the "no reserve" system than under the reserve system. If this higher interest rate is used in calculating each person's implicit fund under assessment financing and in discounting social insurance benefits to the present, the equality between expected present values of premiums and benefits is restored.

II. THE PARADOX DEMONSTRATED

On the macroeconomic level, however, the paradox remains intact. This section will be devoted to a rigorous demonstration of this proposition. Assume that, upon reaching a particular age A , all people enter the labour force and that the number reaching this age year grows at the rate of g percent. Assume that average real wages grow at the rate of h percent and that the interest rate is i percent. Let $t=1+g$, $s=1+h$, and $r=1+i$. Each person works m years then retires. He lives in retirement from $n-m$ years dying at the age of $A+n$. During each year of retirement each person receives a pension equal to the average wage then prevailing among active workers.

After k years (where $k > n$), the total population, $\bar{P}$, will be:

$$(1) \quad \bar{P} = P_0 t^k [(t^m + t^{m-1} + \dots + t) + (1 + t^{-1} + t^{-2} + \dots + t^{-n+m+1})],$$

where P_0 is the size of the retiring cohort at time 0. The first parenthetical expression inside the brackets, multiplied by $P_0 t^k$, gives the total working population; the second parenthetical expression, multiplied by $P_0 t^k$, gives the total retired population.

The total wages any worker receives during his working life, w , will be:

$$(2) \quad w = w_0 s^k (s^{-m} + s^{-m+1} + \dots + s^{-1}),$$

where the worker retires in year k , and w_0 is the wage prevailing in year 0.

During retirement the same worker will receive pensions, p , indicated by:

$$(3) \quad p = w_0 s^k (1 + s + s^2 + \dots + s^{n-m-1}).$$

During his working life, each pension recipient had paid taxes to support pensions. Assuming that taxes were just sufficient to cover pension costs; i.e., that no reserve was accumulated, each worker paid a fraction of his wage, f , equal to the ratio of the retired to the active population. This fraction, f , is equal to the ratio of the second parenthetical expression in equation (1) to the first. Assuming that these