taxes had been invested, as they would be in private insurance or in funded social insurance, they would have a present value, PV_T, at the time the worker retires shown by equation (4).

$$(4) \qquad PV_T = fw_0 s^k (s^{-m} r^m + s^{-m-1} r^{m-1} + \ldots + s^{-1} r).$$

Assuming that at retirement the worker discounts future benefits at the interest rate i, the present value of pensions, benefits PV_B, is shown by equation (5).

$$(5) \qquad PV_B = w_0 s^k = (1 + sr^{-1} + s^2 r^{-2} + \ldots + s^{n-m-1} r^{-n+m+1}).$$

The paradox presented in section I states that $PV_B > PV_T$ when r is below some value. I shall now show that the critical value of r occurs approximately when $i = g + h$; i.e., where the rate of interest equals the sum of population growth and the increase in real wages. To do so it is necessary to establish the precise conditions under which $PV_B > PV_T$. Writing both out in full, moving the numerator of f to the left of the inequality sign, and canceling where possible, the inequality may be written as follows:

$$(6) \quad \frac{(1 + sr^{-1} - s^2 r^{-2} + \ldots + s^{n-m-1} r^{-n+m+1})}{(1 + t^{-1} + t^{-2} + \ldots + t^{-n+m+1})} > \frac{(s^{-m} r^m + s^{-m+1} r^{m-1} + \ldots + s^{-1} r)}{(t^m + t^{m-1} + \ldots + t)}$$

Now, the right and left expressions are equal when $r = st$. In that case the numerator on each side becomes identical, term for term, with the denominator. The derivative of the left(right)-hand expression with respect to r is clearly negative (positive). Consequently for smaller values of r, the inequality holds. For larger values, it does not. When, r, s, and t differ only slightly from unity, the condition that $r = st$ can be approximated by the condition that $i \approx g + h$.

III. Welfare Implications

The results of section II show that if the sum of the rates of growth of per capita wages and of population exceeds the rate of interest, and if the rate of interest equals the marginal rate of time preference and the marginal rate of transformation of present into future goods, then the introduction of some social insurance pensions on a pay-as-you-go basis will improve the welfare position of each person. If saving and. hence, investment and, hence, the rate of growth of income are reduced as the level of social insurance increases, this conclusion does not necessarily follow. If the rate of growth is unaffected, the effective rate of return on premiums paid for such social insurance will exceed the marginal rate of time preference, and, consequently, people in the active labour force would willingly forego some current consumption in order to obtain such returns. Individually they are unable to do so; collectively they can.

If a small trust fund is accumulated, the proceeds from which are invested, the addition to welfare will be smaller than if no fund is accumulated, and in the limiting case of a full reserve, no increase in welfare will occur.

If the rate of interest exceeds the sum of the rate of growth of real wages and the rate of growth of population, then introduction of social