A REEXAMINATION OF THE PURE CONSUMPTION LOANS MODEL*

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I. INTRODUCTION

Professor Samuelson's article (1958) on an exact consumption-loan model of interest led to an interesting controversy. At issue were the determination and properties of interest rates in a dynamic economic system with no capital. One might have thought that these exchanges, between Samuelson (1959) and Lerner (1959) on the one hand, and between Samuelson (1960) and Meckling (1960) on the other, would result, if not in a complete resolution of the disagreements, at least in the emergence of a clear picture of what the issues are and how they might be treated. Unfortunately, such was not the case. The Samuelson-Lerner and the Samuelson-Meckling dialogues leave the reader rather perplexed, as though he had just watched a New Wave film—executed with brilliance, enjoyable while in progress, but not quite clear as to what is happening and never giving one a sense of resolution.

The 1960's have brought an upsurge of interest in capital theory and, more generally, in questions of allocation over time. Now Samuelson's model, even though it has no capital, is of interest to capital theorists because it has many of the features of a model of capital accumulation with decentralized decisionmaking. Since the 1958–60 discussions of this model by Samuelson, Lerner, and Meckling left some questions unanswered, we feel that a restudy of the model is in order. The following is an attempt at such a restudy.

We shall take the liberty of deviating somewhat from Samuelson's original notation.

II. THE MODEL

We shall concentrate throughout most of our discussion on the simpler of Samuelson's two models; namely, the one in which people live for two periods, earning a fixed income in the first and earning nothing in the second. In the first period of his lifetime, a person earns *one* unit of output, where "output" is something usable directly (and exclusively) in consumption, and we do not inquire wherefrom it comes.

The generation which is born at time t will be called generation t, and we shall assume that there are $(1+n)^t$ people in it. Thus, n is the (relative) rate of growth of population, which is assumed constant. Members of generation t are thought of as being alike in all respects,

*Reprinted from *The Journal of Political Economy*, August 1966, vol. LXXIV, No. 4.

**Yale University. We are indebted to Richard Attiyeh, Herbert Scarf, and James Tobin for their comments. This essay is based on research which was supported by the National Science Foundation.