IX. A Three-Period Model.

We have argued that in order to have efficiency there must be someone in the economy who is, at least periodically, a net borrower. Along stationary paths, this must be the case in *every* period. However, there was nothing in that argument to suggest that the net borrower must be a net borrower throughout his (or its) lifetime. Indeed, this role may well change hands over time, which was not the case with the financial intermediary of section VII.

The prime candidate for this state of temporary net borrowership is the consumer himself. To check on this possibility, let us consider the following modification of our model: Assume that people live for three periods rather than two. In the first period of life a person grows up and is educated and therefore earns nothing; in the second period he works and earns one unit of output; in the third he is retired. Every person will now be a net borrower at the end of his first period, a net lender at the end of his second period, and he will be neither a borrower nor a lender at the end of his last period. (We are not using the terms "negative net worth" and "positive net worth" because it is not clear that they are applicable in the present context.) We shall restrict our attention to stationary paths. Along a stationary path, everybody receives the same lifetime consumption profile, say (C^1, C^2, C^3). Feasibility and efficiency now means that the following equation holds:

$$(1+r)C^1+C^2+\frac{C^3}{1+n}=1,$$

which is similar to the feasibility and efficiency equation in the two-period case and is derived in the same way. If we let the (constant) rate of interest along the stationary path be denoted r, then each person's budget constraint is given by

$$(1+n)C^1+C^2+\frac{C^3}{1+r}=1,$$

which leads, once again to $r=n$. In other words, the only rate of interest which can be established along an efficient stationary path is n. Each person is now viewed as maximizing the utility function $U(\bar{C}^1,\bar{C}^2,\bar{C}^3)$ subject to the budget constraint (with n replacing r). The maximization will result in an *optimal* consumption plan, say $(\bar{C}^1,\bar{C}^2,\bar{C}^3)$. This optimal triple must be examined to see if it can be brought about by purely competitive (decentralized) trades.

The optimal consumption plan $(\bar{C}^1,\bar{C}^2,\bar{C}^3)$ will be said to be *competitively attainable* if the following equation holds:

$$\bar{C}^3=(1+n)^2\bar{C}^1.$$

To see how this condition is obtained, consider a person of generation t. When he is young, he borrows an amount $\bar{C}^1$ from members of generation $t-1$, who are in their middle years. Next period, when he is middle-aged and generation $t-1$ is retired, he returns the loan, plus interest. In other words, he returns the amount $(1+n)\bar{C}^1$. Now aggregate borrowing by generation t (when young) is given by the amount $(1+n)^t\bar{C}^1$, and, therefore, total payment by generation t