The aggregate debt of the consumer sector to the fiancial intermediary will grow like a geometric progression, but each consumer by himself will be balancing expenditures and receipts over his lifetime; that is, at the end of his last period of life he will have zero net worth. Under these circumstances, the intermediary may very well be thought of as a privately owned, competitive institution.

X. Efficiency and Infinity

The possible inefficiency (or nonoptimality) of the competitive mechanism, as demonstrated by Samuelson and Diamond, has given rise to a certain amount of speculation, mostly on an informal basis. Many people (including Samuelson [for example, 1958, p. 474; 1959, p. 522] and Diamond [1965, p. 1134]) seem to feel that this phenomenon has something to do with infinity. What apparently leads one to point an accusing finger at "infinity" is the fact that for the standard general equilibrium model (which is finite) we have theorems which tell us that the competitive mechanism always leads to an optimum (and, a fortiori, to efficiency). Nevertheless, the role played by "infinity" in leading the competitive mechanism astray has remained, at best, rather vague. In the present section, we wish to explore this question somewhat more systematically by trying to construct a finite model that resembles the infinite model of the foregoing discussion as closely as possible. As it turns out, inefficiency may well arise in such a finite model.

Consider an economy with m agents and m commodities (where $m>2$). Each agent is both a consumer and a producer. Let C_i^j be the amount of commodity j *consumed* by agent i, and let Q_i^j be the amount of commodity j *produced* by agent i. Agent i (for $i=1, 2, \ldots, m-1$) is assumed to desire, for consumption, only two commodities: commodity i and commodity $i+1$. (Agent m is assumed to desire commodity m and commodity 1.) Thus, agent i's utility function is given by

$$U_i=U(C_i^i, C_i^{i+1})i=1, \ldots, m-1, U_m=U(C_m^m, C_m^1),$$

where the function U is common to all. As for production possibilities we assume that agent i can produce commodities i and $i+1$ (with agent m producing commodities m and 1), but that he has a relative advantage in the production of commodity i. More specifically, we shall assume that agent i can produce any combination of Q_i^i *and* Q_i^{i+1} satisfying

$$Q_i\geqq 0,\, Q_i^{i+1}\geqq 0,\, Q_i^i+\frac{Q_i^{i+1}}{1-\delta}\leqq 1,$$

where δ is some real number satisfying $0<\delta<1$.[10]

Recall for a moment the infinite model of Section VI (with durable output). That model is easily shown to be mathematically equivalent to a model in which population is stationary, while the storing of out-

[10] We shall, from now on, neglect to write separate expressions for the case $i=m$. Let us agree, therefore, that whenever $i=m$, $i+1$ is simply 1.