put is subject to *depreciation* at some constant rate, say δ. But the model which we have described in the foregoing paragraph is an exact finite analogue of this infinite model with depreciation. For, by chopping off a finite segment of the infinite model and then tying the two ends together to form a closed loop, one obtains the model that is being discussed here.

Let us, therefore, investigate the efficiency of the competitive mechanism in our finite model, to which we shall henceforth refer, for short, as the *closed-loop model.* This investigation turns on whether or not intermediation is permitted.

Case a: No intermediation.—Under the assumption that intermediation is altogether absent, it is evident that the closed-loop model is inefficient. For, without intermediation, an agent who wishes to trade, say, x units of commodity j for y units of commodity k, must find, in order to be able to make the trade, an agent who wishes to trade y units of comodity k for x units of commodity j. This is, of course, the well-known "double coincidence of wants" (see, for example, Samuelson, 1964, p. 51). Now, in the closed-loop model, things are tailored in such a way that whatever trade an agent might wish to engage in, at whatever prices, he can never find another agent wishing to engage in the same trade on the opposite side. In other words, the double coincidence of wants never occurs. In the absence of intermediation, therefore, each agent must satisfy all his wants under complete autarky, and the doctrine of comparative advantage tells us immediately that this is inefficient. More formally, the decentralized solution of the closed-loop model in the absence of intermediation is given by

$$\left.\begin{array}{l} C_i^i=Q_i^i \\ C_i^{i+1}=Q_i^{i+1} \end{array}\right\} \text{ for } i=1, \ldots, m,$$

where(Q_i^i, Q_i^{i+1}) is chosen so as to maximize $U(Q_i^i, Q_i^{i+1})$, subject to

$$Q_i^i+\frac{Q_i^{i+1}}{1-\delta}=1,\ Q_i^i \geqq 0,\ Q_i^{i+1} \geqq 0.$$

Under the assumption that $\delta>0$, this solution is inefficient. The economy can produce more of every commodity.

Now let us go back for a moment to the model of section VI. The inefficiency which beset the economy of that section is precisely the inefficiency which now besets our closed-loop model. All the arguments (indeed, the very words) of section VI are applicable to the closed-loop model, with only minor changes which have to do with substituting depreciation for population growth.

Case b: The money economy.—The necessity for a double coincidence of wants disappears, as is well known, as soon as barter is abandoned in favor of the money economy. Thus, one might expect the introduction of "the contrivance of money" to cure the aforementioned inefficiency, just as it cured the inefficiency of the model of section VI. Such, indeed, is the case. In the money economy, agent i will produce commodity i exclusively, sell some of it to agent $(i-1)$, and then use the money which he receives in return to buy some of agent $(i+1)$'s output. More formally let p_i be the money price of