

commodity i . Then, the equilibrium of the money economy is described by

$$\begin{aligned} p_i &= p_j \text{ for all } i \text{ and } j, \\ Q_i &= 0 \text{ if } i \neq j, \\ &= 1 \text{ if } i = j, \end{aligned}$$

and (C_i^i, C_i^{i+1}) chosen so as to maximize $U(C_i^i, C_i^{i+1})$, subject to $C_i^i + C_i^{i+1} = 1$, $C_i^i \geq 0$, $C_i^{i+1} \geq 0$.

This equilibrium is efficient, and it is precisely analogous to the equilibrium in the money economy that was discussed in section VIII. People "buy" money in return for goods which they produce, and then "sell" it for goods which they want. Basically, money here provides intermediation. Agent i sells to agent $(i-1)$, but he is being paid in terms of output of agent $(i+1)$.

There are several ways in which money can be introduced into the closed-loop model. The simplest is as follows: Think of agent 1 selling a promissory note to agent 2, who, in turn, proceeds to sell it to agent 3, and so on. The note travels around the loop until it reaches agent m , who redeems it from agent 1. All other agents, however, accept agent 1's note not because they are interested in agent 1's output, but simply because the note is *negotiable*. In other words, agent 1's note will assume the role of money.

Case c: The ordinary general equilibrium model.—The closed-loop model is, after all, a finite general equilibrium model, and for the latter we have theorems concerning optimality and efficiency. What role, then, does intermediation play in the standard general equilibrium theory? The answer to this question is straightforward: In all of general equilibrium theory it is assumed (mostly implicitly) that a *central clearing house* exists, through which trades are channeled. Only with such a central clearing house is it possible to define competitive equilibrium in terms of aggregate excess demands alone. If a central clearing house were to be introduced in the closed-loop model, competitive behavior would lead to efficiency, as may be verified directly (without appealing to general theorems). But, the central clearing house is obviously an intermediary. It is, in fact, precisely the analog, in the closed-loop model, of our negative net worth intermediary of section VII.

XI. CONCLUDING REMARK

Before closing, we would like to add a brief comment concerning the relation of the topics we have been discussing here to the Modigliani-Brumberg (1955) "life-cycle" theory of saving. Modigliani and Brumberg have postulated that aggregate saving can be explained by the interaction of individual saving for retirement and changes in the population structure. It is interesting to note that under the Modigliani-Brumberg assumptions of a zero rate of interest and an exponentially growing population, providing for old age in a way which results in positive aggregate saving is inefficient.¹¹

¹¹ We are indebted to James Tobin for this observation.