and A_l = perpetual future consumption stream that l can enjoy.

Two identities are implicit in these definitions:

(1) $Y_l = C_l + S_l$, and

(2) $i_l \Delta S_l = \Delta A_l$.

The identity (2) defines the rate at which saving produces a future consumption stream. We assume a utility function

(3) $U_l = U_l (C_l, A_l)$,

which reflects l's present valuation of current consumption and consumption in the future. He maximizes his utility function subject to his income and interest constraints. This is equivalent to maximizing the Lagrangean expression

(4) $\phi = U_l (C_l, A_l) - \lambda (Y_l - C_l - S_l) - \mu (i_l \Delta S_l - \Delta A_l)$,

which has the first-order maximum conditions

(5) $\dfrac{\partial U_l}{\partial C_l} + \lambda = 0, \quad \lambda - \mu i_l = 0, \quad \text{and} \quad \dfrac{\partial U_l}{\partial A_l} + \mu = 0.$

Therefore,

(6) $\dfrac{\partial U_l}{\partial C_l} = -\mu i_l \quad \text{and} \quad \dfrac{\partial U_l}{\partial A_l} = -\mu$, and so

(7) $\dfrac{\dfrac{\partial U_l}{\partial C_l}}{\dfrac{\partial U_l}{\partial A_l}} = i_l, \quad \text{or} \quad \dfrac{\partial U_l}{\partial C_l} = i_l \dfrac{\partial U_l}{\partial A_l}.$

A change in taxation is a change in disposable income, part of which will change consumption, part saving. Thus,

(8) $\Delta Y_l = \Delta C_l + \Delta S_l$,

and the change in l's utility is

(9) $\Delta U_l = \dfrac{\partial U_l}{\partial C_l} \Delta C_l + \dfrac{\partial U_l}{\partial A_l} \Delta A_l$,

neglecting higher order terms on the grounds that they will be of the second order of smalls. (9) is equivalent to